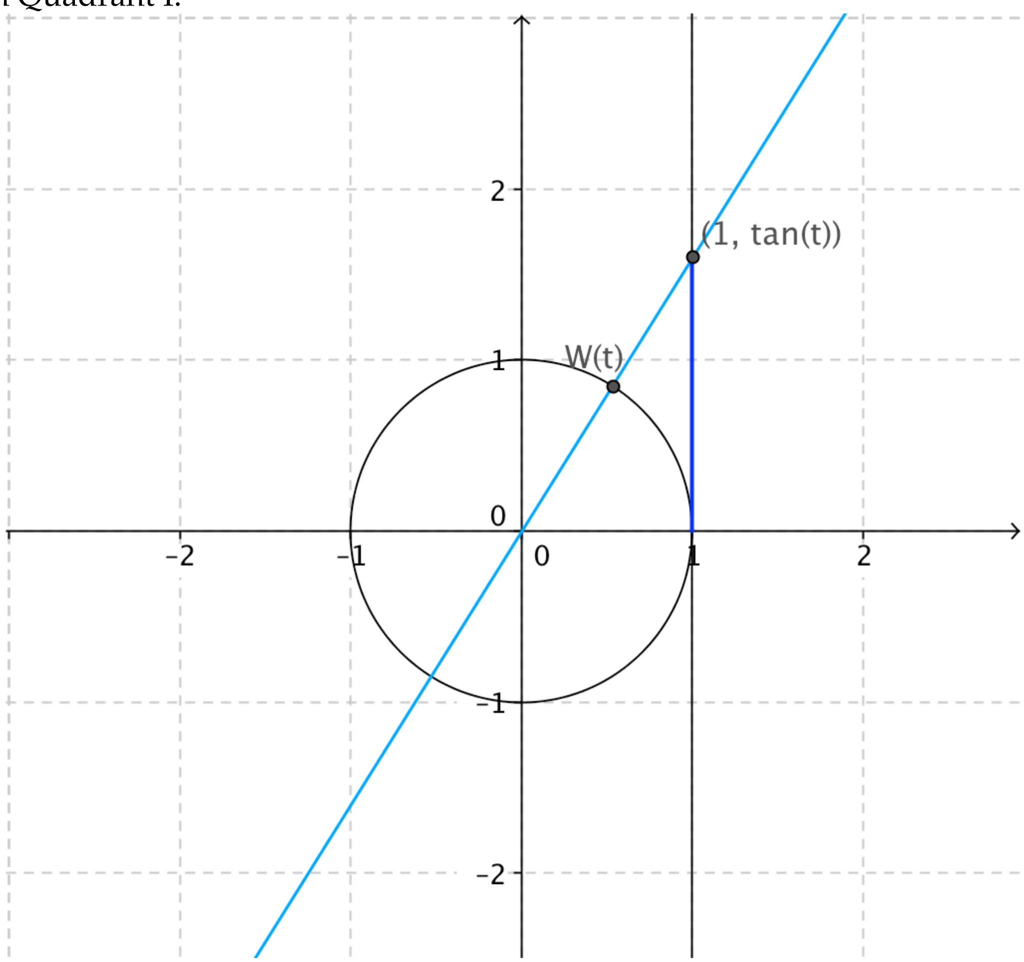
The tangent function

Define a new function, tan(t), [short for tangent(t)] by using the wrapping function W(t) which we have already used when studying sine and cosine.

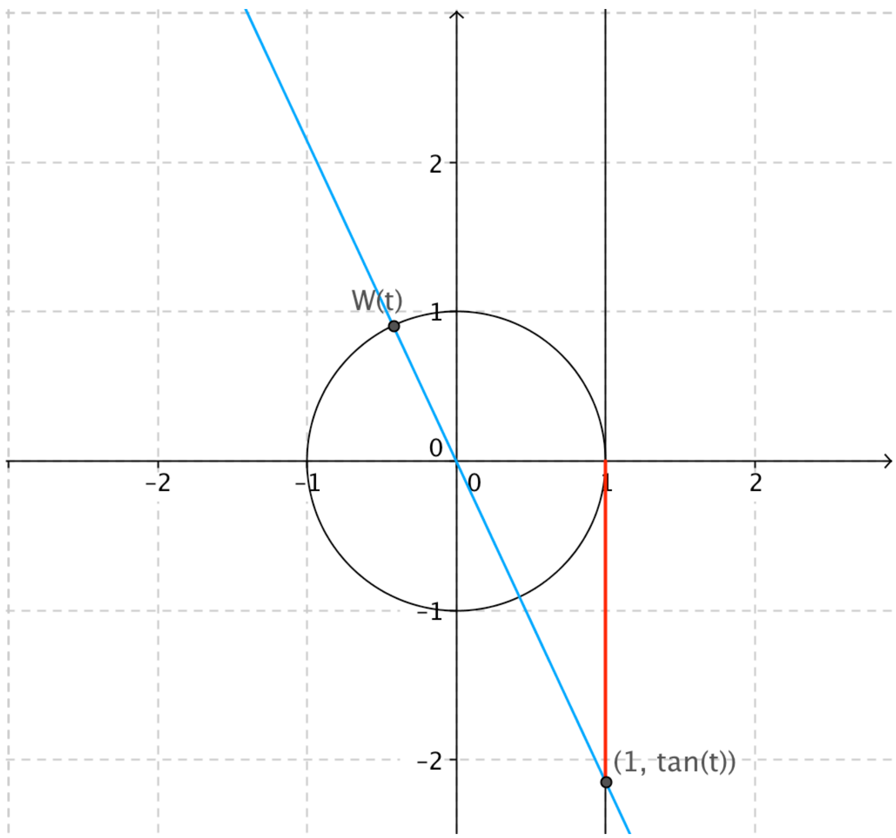
Definition of tan(t): On the unit circle, construct line l tangent to the circle at the point (1,0). Find the point W(t) on the unit circle and construct the line through the origin and W(t) . Now find the intersection of this line and the tangent line l. We define tan(t) to be the second coordinate of this point of intersection.

While the definition can seem cumbersome when written down this way, it is actually quite easy to find tan(t). Here are two different cases, one with W(t) in Quadrant I and the other with W(t) in Quadrant II.

W(t) in Quadrant I:



W(t) in Quadrant II



Note that if W(t) is in Quadrant I, then tan(t) is positive and if W(t) is in Quadrant II, tan(t) is negative.

Using the unit circle and the tangent line in the diagram on the following page(s), answer the following:

1. What is $\tan(0)$?

2. What is $\tan\left(\frac{\pi}{4}\right)$?

3. What is $\tan\left(\frac{\pi}{6}\right)$?

4. What is $\tan\left(\frac{\pi}{3}\right)$?

5. Explain why $\tan\left(\frac{\pi}{2}\right)$ is undefined. (Does not exist.)

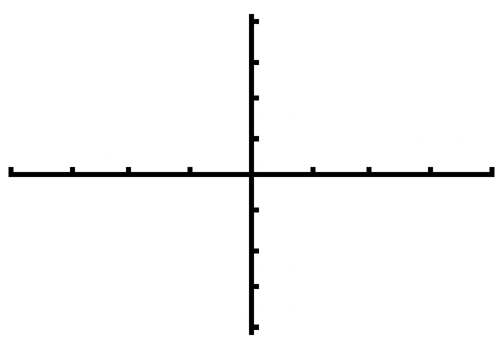
6. Demonstrate why $\tan(t+\pi)=\tan(t)$ for all t for which $\tan(t)$ exists.

7. In what quadrant(s) is $W(t)$ if $\tan(t)$ is positive?

8. In what quadrant(s) is $W(t)$ if $\tan(t)$ is negative?

9. On the axes provided, sketch the graph of $f(t)=\tan(t)$ for $-2\pi \le t \le 2\pi$.

Note that each the "tics" on the X-axis are $\pi/2$ units apart and are 1 unit apart on the Y-axis.

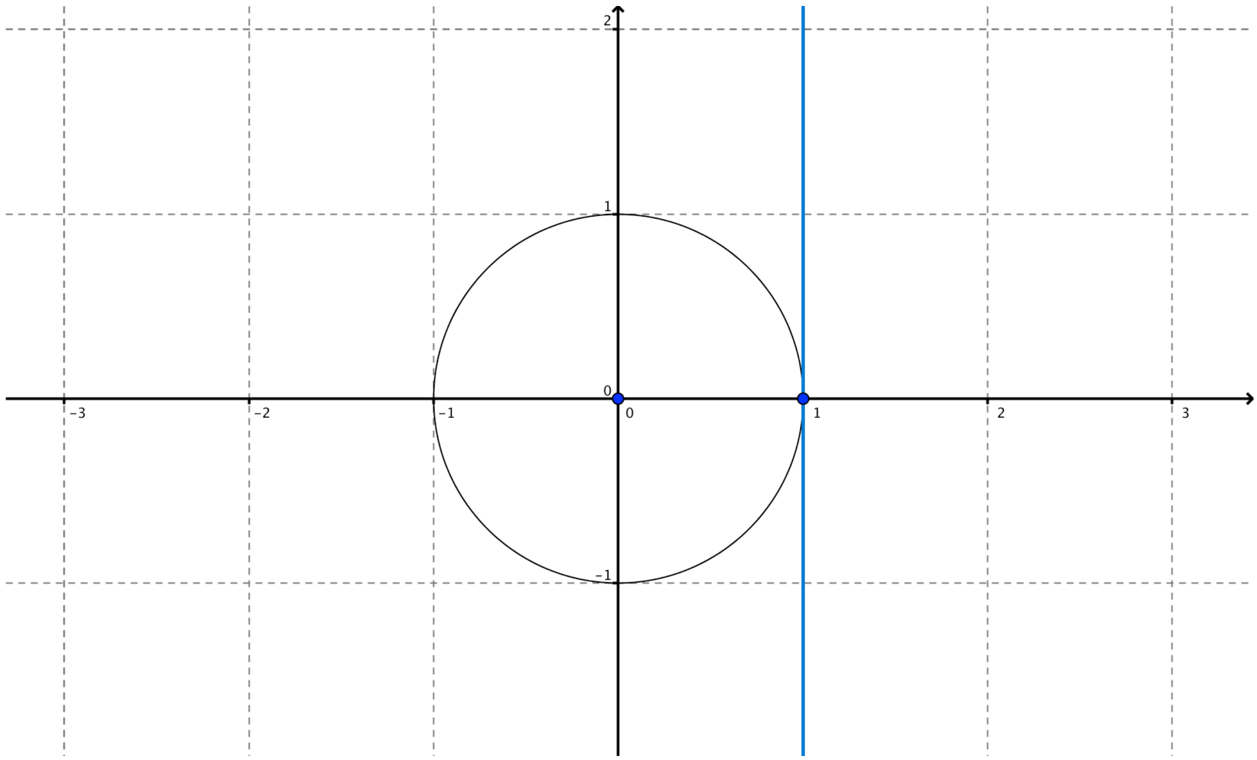


10. What is the Range of tan?

11. What is the Domain of tan?

12. What is the period of tan?

13. By using similar triangles, prove that $\tan(t)=\dfrac{\sin(t)}{\cos(t)}$ whenever $\cos(t)\neq 0$.



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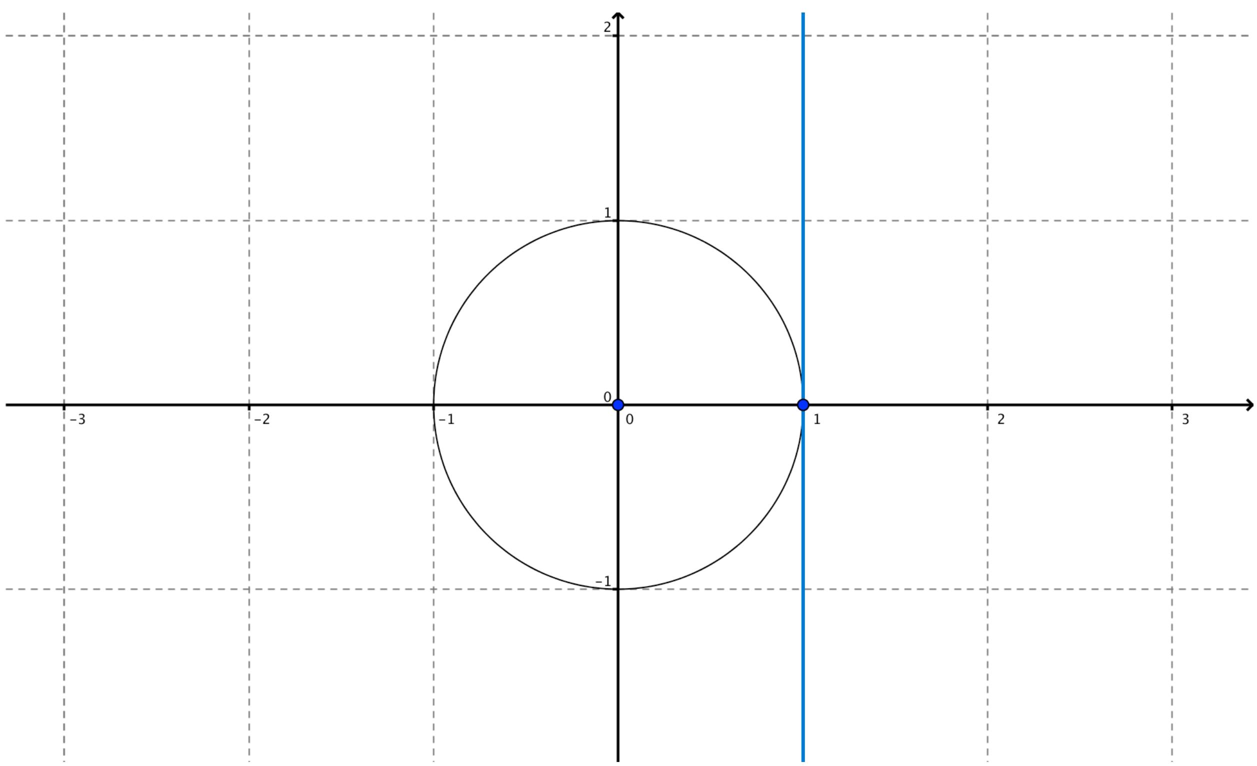

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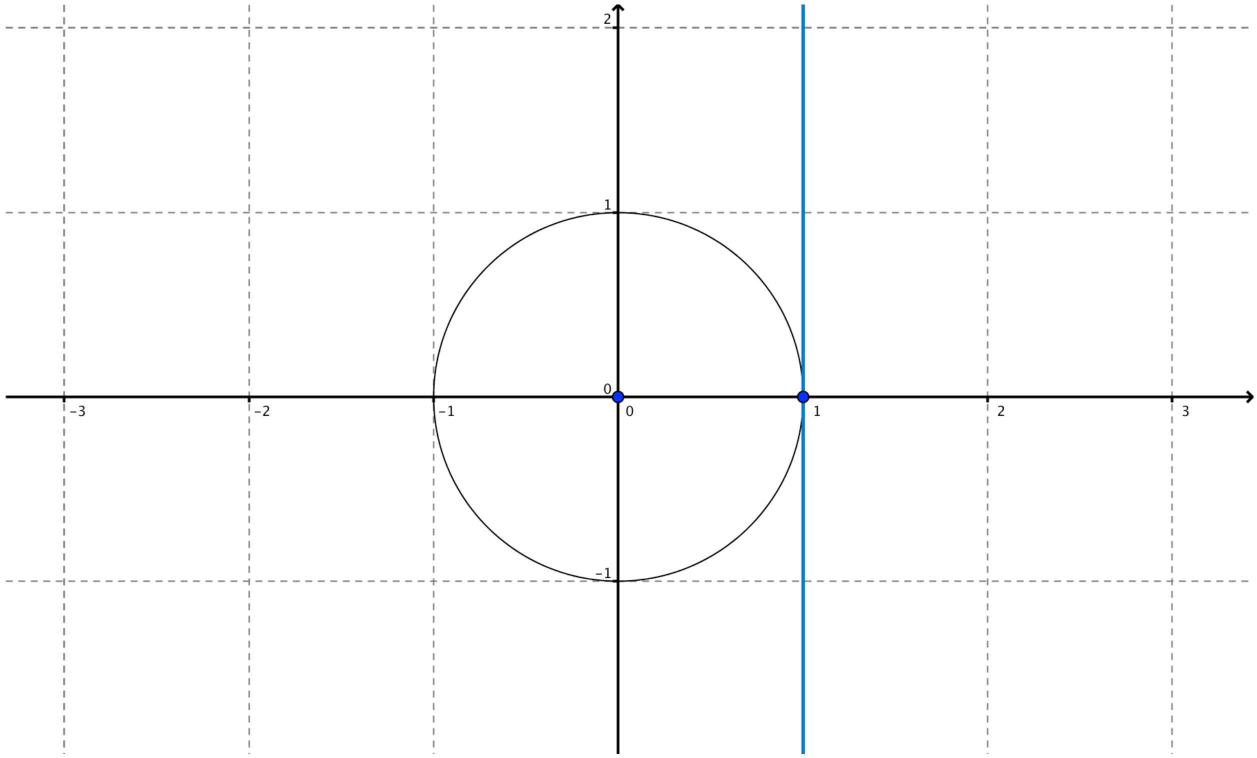
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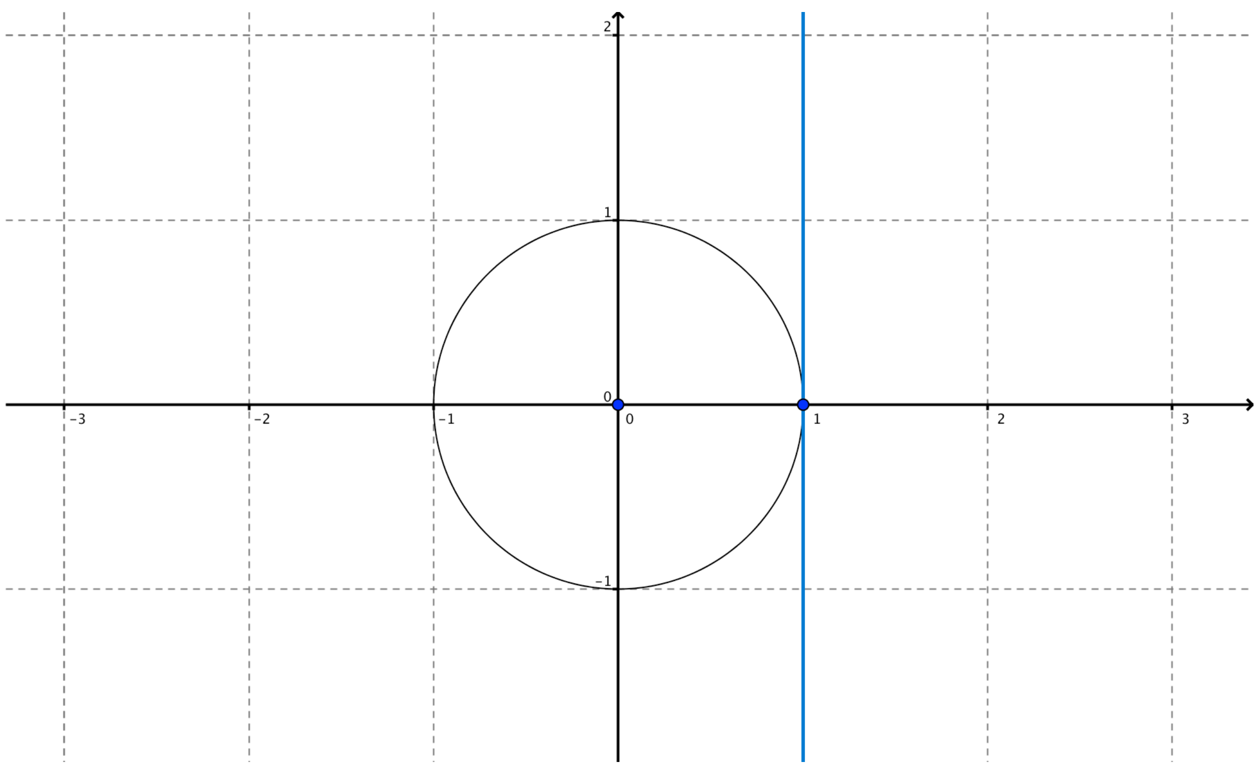
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